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$$\begin{aligned}
& \text{When } n=2, \frac{b-d/dx}{a^2+b^2} \left[\frac{1}{(a^2+b^2)^{\frac{1}{2}}} \cos(ax-\theta) \right] \\
&= \frac{1}{a^2+b^2} \left[\frac{b \cos(ax-\theta) + a}{\sqrt{a^2+b^2}} \sin(ax+\theta) \right] \\
&= \frac{1}{a^2+b^2} \left[\cos\theta \cos(ax-\theta) + \sin\theta \sin(ax-\theta) \right] = (a^2+b^2)^{-\frac{1}{2}} \cos(ax-2\theta).
\end{aligned}$$

$$\begin{aligned}
& \text{When } n=3, \therefore \frac{b-d/dx}{a^2+b^2} \left[\frac{1}{(a^2+b^2)^{\frac{1}{2}}} \cos(ax-\theta) \right] \\
&= \frac{1}{a^2+b^2} \left[\frac{b \cos(ax-2\theta) a \sin(ax-2\theta)}{\sqrt{a^2+b^2}} \right] = (a^2+b^2)^{-\frac{3}{2}} \cos(ax-3\theta).
\end{aligned}$$

This method holds for $(n-1)$ terms.

$$\begin{aligned}
& \text{Hence } y = e^{-bx} (c_1 + c_2 x + c_3 x^2 + c_4 x^3 + \dots c_n x^{n-1}) \\
& \quad + (a^2 + b^2)^{-\frac{1}{2}n} \cos[ax - n \cot^{-1}(b/a)].
\end{aligned}$$

SECOND SOLUTION.

$$\begin{aligned}
& \text{Put } D = \frac{d}{dx}, \text{ then we have } \frac{1}{(D+b)^n} \cos ax = \frac{(D-b)^n}{(D^2-b^2)^n} \cos ax \\
&= \frac{(-1)^n (b-D)^n}{(a^2+b^2)^n} \cos ax.
\end{aligned}$$

$$\begin{aligned}
& \text{The numerator} = (-1)^n \left[b^n \cos ax + n a b^{n-1} \sin ax \right] - \frac{n(n-1)}{2!} a^2 b^{n-2} \cos ax \dots] \\
&= (-1)^n \cos[ax - n \cot^{-1}(b/a)]
\end{aligned}$$

The value of y is the same as in I.

Also solved by WILLIAM HOOVER, G. B. M. ZERR, and the PROPOSER.

Professor Landis remarks that the exponent of a^2+b^2 is given $\frac{1}{2}n$ in Johnson's *Differential Equations* (problem 17, page 122) instead of the correct result $-\frac{1}{2}n$.

122. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

$$\text{Solve the differential equation } (y-x)\sqrt{1+x^2} \frac{dy}{dx} = n(1+y^2)^{\frac{1}{2}}$$

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

$$\text{Let } x = \tan \theta, y = \tan \phi, \text{ then } \sin(\phi - \theta) d\phi = n d\theta \dots (1).$$

$$(\tan \varphi - \tan \theta) \sqrt{1 + \tan^2 \theta} \frac{\sec^2 \varphi d\varphi}{\sec^2 \theta d\theta} = n(1 + \tan^2 \varphi)^{\frac{3}{2}} = n \sec^3 \varphi \dots (2).$$

$$(\tan \varphi - \tan \theta) d\phi = n \sec \phi \sec \theta d\theta \dots (3).$$

$$\frac{\sin \varphi \cos \theta - \cos \varphi \sin \theta}{\cos \varphi \cos \theta} d\varphi = \frac{n d\theta}{\cos \varphi \cos \theta} \dots (4). \quad n \frac{d\theta}{d\varphi} = \sin(\varphi - \theta) \dots (5).$$

$$\text{Let } \omega = \varphi - \theta, \text{ then } \frac{d\omega}{1 - (\sin \omega / n)} = d\varphi \dots (6).$$

$$\text{Whence } \cot \left[\frac{\pi}{4} - \frac{\varphi - \theta}{2} \right] = n(\varphi + c) \dots (7).$$

$$\cot \left[\frac{1}{2} \pi - \frac{1}{2} (\tan^{-1} y - \tan^{-1} x) \right] = n \tan^{-1} y + c_1, \text{ or, let}$$

$$y = \frac{x-z}{1+xz}; \text{ then } \frac{z}{[1+z^2][z+n\sqrt{1+z^2}]} \frac{dz}{dx} - \frac{1}{1+x^2} = 0,$$

in which the variables are separated.

II. Solution by F. ANDEREGG, A. M., Professor of Mathematics, Oberlin College, Oberlin, O.

$$\text{Let } x = \tan \theta, y = \tan \varphi, \text{ then } (\tan \varphi - \tan \theta) \sec \theta \frac{\sec^2 \varphi}{\sec^2 \theta} = n \sec^3 \varphi.$$

$$\therefore \sec(\varphi - \theta) = n \frac{d\theta}{d\varphi}.$$

$$\text{Let } \omega = \varphi - \theta, \text{ then } \frac{d\theta}{d\varphi} = 1 - \frac{d\omega}{d\varphi}. \quad \therefore \sin \omega = n - n \frac{d\omega}{d\varphi}.$$

$$\therefore \frac{n d\omega}{n - \sin \omega} - d\varphi = 0.$$

By the formulas given in the text books, if $n > 1$, numerically,

$$\frac{2n}{\sqrt{n^2 - 1}} \tan^{-1} \frac{n \tan \frac{1}{2} \omega - 1}{\sqrt{n^2 - 1}} = \varphi + c_1.$$

$$\therefore \frac{2n}{\sqrt{n^2 - 1}} \frac{n \tan \left(\frac{1}{2} \tan^{-1} \frac{y-x}{1+xy} \right) - 1}{\sqrt{n^2 - 1}} = \tan^{-1} y + c_1; \text{ if } n < 1, \text{ numerically.}$$

$$\frac{1}{\sqrt{1-n^2}} \log \frac{n \tan \frac{1}{2} \omega - 1 - \sqrt{1-n^2}}{n \tan \frac{1}{2} \omega - 1 + \sqrt{1-n^2}} = \varphi + c_2.$$

$$\therefore \frac{1}{\sqrt{1-n^2}} \log \frac{n \tan \left(\frac{1}{2} \tan^{-1} \frac{y-x}{1+xy} \right) - 1 - \sqrt{1-n^2}}{n \tan \left(\frac{1}{2} \tan^{-1} \frac{y-x}{1+xy} \right) - 1 + \sqrt{1-n^2}} = \tan^{-1} y + c_2.$$

If $n=1$, $\frac{d\omega}{1-\sin\omega} - d\varphi = 0$.

$$\therefore \frac{d\omega}{\sin^2 \frac{1}{2} \omega + \cos^2 \frac{1}{2} \omega - 2 \sin \frac{1}{2} \omega \cos \frac{1}{2} \omega} - d\varphi = 0.$$

$$\therefore \frac{d\omega \sec^2 \frac{1}{2} \omega}{(\tan \frac{1}{2} \omega - 1)^2} - d\varphi = 0. \quad \therefore \frac{2d(\tan \frac{1}{2} \omega - 1)}{(\tan \frac{1}{2} \omega - 1)^2} - d\varphi = 0.$$

$$\therefore \frac{2}{1 - \tan \frac{1}{2} \omega} = \varphi + c_2. \quad \therefore 1 + \frac{\tan \frac{1}{2} \omega + 1}{1 - \tan \frac{1}{2} \omega} = \varphi + c_2.$$

$$\therefore \tan \left(\frac{1}{2} \omega + \frac{1}{4} \pi \right) = \varphi + c'^2.$$

$$\therefore \tan \left[\frac{\pi}{4} + \frac{1}{2} \tan^{-1} \frac{y-x}{1+xy} \right] = \tan^{-1} y + c'_2; \text{ if } n = -1,$$

$$\frac{d\omega}{1+\sin\omega} - d\varphi = 0, \text{ and the result is } \tan \left[\frac{\pi}{4} - \frac{1}{2} \tan^{-1} \frac{y-x}{1+xy} \right] = \tan^{-1} y + c.$$

Also solved by *L. C. WALKER*, *H. C. WHITAKER*, and *G. B. M. ZERR*.

123. Prize Problem. Proposed by *B. F. FINKEL*, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

Find in finite terms, the value of $\int_0^{\frac{1}{2}\pi} \log \tan \phi d\phi$.

Solution by *W. H. ECHOLS*, Professor of Mathematics, University of Virginia, Charlottesville, Va.

$$\text{Put } \tan z = x. \quad \text{Then } \int_0^{\frac{1}{2}\pi} \log(\tan z) dz = \int_0^1 \frac{\log x}{1+x^2} dx,$$

$$= \int_0^1 \log x (1 - x^2 + x^4 - x^6 + \dots) dx.$$

$$\text{Differentiating, respecting } a, \quad \int_0^1 x^a dx = \frac{1}{a+1}.$$

$$\therefore \int_0^1 x^a \log x dx = -\frac{1}{(a+1)^2}.$$